Only attempt this if you are done with the worksheet and have additional time. For your reference, I've attached some notes on basic differential equations on the last page of this handout. We will go over this together at the end of today's module.

Problem 1. The number of cells in a T25 flask is growing at a rate of $\alpha \exp(\lambda t)$, where t represents the time elapsed since the initial time.

(a) Write a differential equation to model the rate at which the cells are dividing. Use the function $F(t)$ to denote the number of cells at any given time.

$$\frac{dF}{dt} = \alpha \exp(\lambda t) \qquad (1)$$

Rate of change in F with respect to t

Given Rate

(b) At $t = 0$, there are ϕ cells in the flask. Use the equation you wrote in part (a) to find the number of cells present at time $t = \delta$.

Let's first solve equation (1) for $F(t)$. We can use the separation of variables method. First, we move the terms over so that the left side is in terms of F, and the right side is in terms of t.

$$dF = \alpha \exp(\lambda t)\, dt \qquad (2)$$

Now, integrate both sides:

$$\int dF = \int \alpha \exp(\lambda t)\, dt$$

$$\int dF = \alpha \int \exp(\lambda t)\, dt$$

$$F = \frac{\alpha \exp(\lambda t)}{\lambda} + C \qquad (3)$$

constant

Continued...

We are given the initial value that at $t=0$, $F(t)=\phi$.
Substitute this into equation (3) to solve for C.

$$\phi = \frac{\alpha \exp(\lambda \cdot 0)}{\lambda} + C$$

$$\phi = \frac{\alpha}{\lambda} + C$$

$$C = \phi - \frac{\alpha}{\lambda} \qquad (4)$$

Combine equations (3) and (4):

$$F = \frac{\alpha \exp(\lambda t)}{\lambda} + \phi - \frac{\alpha}{\lambda}$$

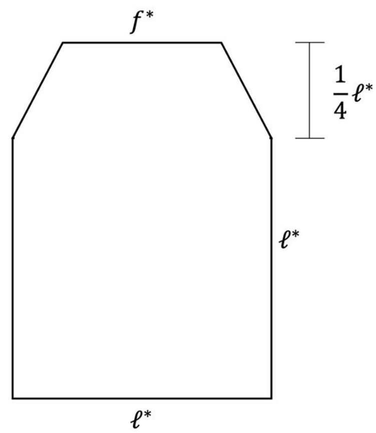
$$F = \frac{\alpha \exp(\lambda t) - \alpha}{\lambda} + \phi$$

$$F = \frac{\alpha[\exp(\lambda t) - 1]}{\lambda} + \phi \qquad (5)$$

Substitute δ for t to find the number of cells at $t=\delta$:

$$\boxed{F = \frac{\alpha[\exp(\lambda \delta) - 1]}{\lambda} + \phi} \qquad (6)$$

(c) Assume all cells are adhered to the bottom of the flask and that each of the cells can be modelled as a 2D circle with radius R^* in the order of μm. The flask has the dimensions shown in the image below, where f^* and l^* are known constants. Using this information and your answer from part (b), calculate the cell confluency at time $t = \delta$.



By definition, cell confluency is given by the equation:

$$\text{Cell Confluency} \equiv \frac{\text{Area occupied by Cells}}{\text{Total Surface Area of Flask}}$$

Since we are given that the radius of each cell is R^*, the area of <u>ONE</u> cell is given by

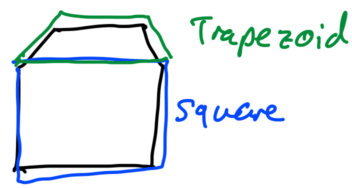
$$A_{\text{one cell}} = \pi (R^*)^2 \qquad (7)$$

Multiply equation (7) by the total number of cells at $t=\delta$, as given by equation (6) to get the total area occupied by all cells:

$$A_{\text{total cells}} = \pi (R^*)^2 \left[\frac{\alpha \left[\exp(\lambda\delta) - 1 \right]}{\lambda} + \phi \right] \qquad (8)$$

Continued...

Now, let's calculate the total area of the flask. We can break it down into a square component and a trapezoidal component.



The area of the square is

$$A_{sq} = (\ell^*)^2 \qquad (9)$$

The area of the trapezoid is

$$A_{tr} = \left(\frac{f^* + \ell^*}{2}\right)\left(\frac{1}{4}\ell^*\right)$$

$$A_{tr} = \ell^*\left(\frac{f^* + \ell^*}{8}\right) \qquad (10)$$

So, the total area of the flask is given by the sum of eq (9) and (10):

$$A_{fl} = (\ell^*)^2 + \ell^*\left(\frac{f^* + \ell^*}{8}\right)$$

$$A_{fl} = \ell^*\left[\ell^* + \left(\frac{f^* + \ell^*}{8}\right)\right] \qquad (11)$$

By definition, the cell confluency at time $t = \delta$ is given by dividing eq (8) by eq (11):

$$\boxed{\text{Cell Confluency at } t = \delta = \frac{\pi (R^*)^2 \left[\frac{\alpha [\exp(\lambda \delta) - 1]}{\lambda} + \phi\right]}{\ell^*\left[\ell^* + \left(\frac{f^* + \ell^*}{8}\right)\right]}} \qquad (12)$$

Solving Basic Differential Equations Through Separation of Variables

A separable equation is a 1st order differential equation of the form:

$$\frac{dy}{dx} = f(x)g(y) \qquad (1)$$

Use the following steps to solve the equation:

1. Manipulate the equation so that the left side is a function of y and the right side is a function of x.

$$\frac{dy}{g(y)} = f(x)\,dx \qquad (2)$$

2. Integrate both sides (don't forget to add a $+C$)

3. Solve for y if possible and C if given an initial value.